Characterization of all Robust PD-based PSSs : an Interval Arithmetic Approach

M. Soliman
Electrical Power &Machines department, Faculty of Engineering,
Benha University, 108 Shoubra St., B.O. 11241, Cairo, Egypt.
Email: msoliman_28@yahoo.com

Abstract— **Design of conventional power system stabilizers (PSSs) is load-dependent and thus has to be adjusted at every operating condition. An interval arithmetic (IA) based approach is proposed, to rigorously address load uncertainties associated with the design of PSS. The proposed approach characterizes the set of all robust stabilizing PSSs computed for a single-machine infinite-bus system. A robust PSS can properly function over the full range of operating conditions. An interval plant transfer function is determined over the operating range where upper and lower bounds of the coefficients are precisely computed. Using a proportional-derivative (PD) PSS, an interval characteristic polynomial for the closed loop system is generated. Interval Routh-Hurwitz array is developed to determine the boundaries of robust stability region in K_p-K_d plane via IA computation. Thereafter, the results obtained by interval Routh array are relaxed using degenerate interval array with an image-set polynomial of the plant where the boundaries of the robust stability region is exactly computed. Simulation results confirm the effectiveness of a sample controller, which lies within the solution set, as it is applied to the original nonlinear system model under wide loading conditions.**

Index Terms—**Interval arithmetic, power system stabilizer, PD control, Routh-Hurwitz array, robust stability.**

I. Introduction

Power generators are conventionally equipped with automatic voltage regulators (AVRs) to control their terminal voltages and improve dynamic stability limits [1-3]. Unfortunately, AVRs may add negative damping and adversely affect the transient stability. Conventional power system stabilizers (CPSS) are widely used in electric power system utilities to improve the stability by adding phase lead to the system at a certain operating point (*nominal plant model*) using phase lead compensation techniques [3]. Therefore, conventional fixed-parameter PSS may fail to maintain system stability over wide range of operating conditions or at least leads to a degraded performance once the deviation from the nominal point becomes significant. As a result, design of robust PSSs becomes a priority to cope with uncertainties imposed by constantly variation in operating points. To robustify the performance of a PSS, the design tactic must account for uncertainties. Robust PSS design has been extensively addressed in power system control literature [4-8]. Among various techniques available in robust PSS design literature, H_∞ [4], quantitative feedback theory (QFT) [5], Kharitonov theorem [6], linear matrix inequalities (LMIs) [7] and evolutionary algorithms [8]. In [4], a frequency bound of the system is computed, then a controller is designed for worst-case frequency. Unfortunately, such design requires exhaustive search and results in a higher order controller. In [5], parametric uncertainty in power systems has been handled using QFT, where controller synthesis is reduced to solve a nonlinear optimization problem. In [6], power system uncertainties have been addressed through an interval polynomial where Kharitonov theorem is applied to compute only the gain of a phase lead compensator. In [7], a linear fractional transformation (LFT) is suggested and then an LMI technique is applied to find 4^{th} order controller. An optimal multiobjective design of robust PSS using genetic algorithms is considered in [8]. Obviously, robust PSS design involves three basic issues regarding uncertainty modeling, controller order and solution algorithm. IA has been extensively applied to power system load flow studies [9-10]. Further, AI-based state estimation of power system with network uncertainty is recently considered in [11]. This note explores the analysis and design of a robust PSS via IA. Power system with uncertainty in operating point resembles a parametric model whose parameters are assumed to vary within real compact intervals. As result, interval techniques turn out to be particularly suitable and appear to be a logical direction. Up to our knowledge, design of robust PSS via IA techniques, as proposed here, is a novel approach.

In this paper, the interval model for a single-machine infinite-bus system is developed while active and reactive powers vary within physical intervals, *i.e.*, $P \in [\underline{P} \quad \overline{P}]$, $Q \in [\underline{Q} \quad \overline{Q}]$. Characterization of all robust stabilizing PD-based PSSs is carried out using Routh-Hurwitz array for an interval characteristic polynomial and an image-set characteristic polynomial. The design via IA techniques has the following advantages: (i) it is based on interval arithmetic which is parallel to conventional arithmetic; (ii) the control design is intuitive since it is based on Routh-Hurwitz criterion; (iii) two characterizations of all robust stabilizing PD-based PSSs are considered.

978-1-4799-1303-9/13/$31.00 ©2013 IEEE

The paper is organized as follows. Section II describes how an interval open-loop transfer function is derived. In Sec. III, a mathematical review of interval analysis is presented. In Sec. IV, PSS synthesis via IA is developed. Simulation results are considered in Sec. V. Section VI concludes this work.

II. Problem Formulation

Although a considerable research is being done in synthesizing PSS for multimachine power system [8], no definitive results have been applied in the field. The design is still done on the basis of a single-machine infinite-bus system [2]. The linearized model of such system is described by a block diagram as proposed by deMello and Concordia [1] and it is shown in Figure 1. The model parameters namely $k_1 - k_6$ are load-dependent and thus they have to be updated at each operating point. These parameters are functions of the operating point that is fully described by both active and reactive powers (*P, Q*). In [6], these parameters are expressed as explicit functions in (*P* &*Q*) of a generator. These functions play an essential directive in the sequel. Consequently, the open loop transfer function $G_p(s) = \Delta\omega(s)/U_{pss}(s)$ is in turn load-dependent, and hence it is more convenient to accomplish the design. Open loop transfer function of the linearized has the following general form at any operating point:

$$G_p(s) = \frac{b_1 s}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \tag{1}$$

The coefficients a_0, a_1, a_2 and b_1 vary according to a vector ρ which consists of two independent quantities (*P* and *Q*), *i.e.,* $\rho = [P \quad Q]^T$ while a_3 and a_4 are always constant and independent of machine loading.

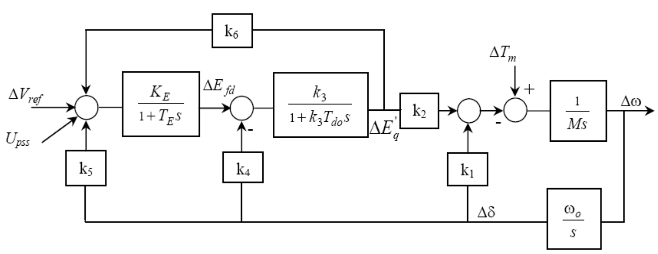

Figure 1 Block diagram of the linearized model [1]

The vector ρ takes values in a *rectangular* whose vertices are given as follows:

$$K_m = \{(\underline{P} \quad \underline{Q}), (\underline{P} \quad \overline{Q}), (\overline{P} \quad \underline{Q}), (\overline{P} \quad \overline{Q})\} \tag{2}$$

Simply, any change in *P* and/or *Q* leads to corresponding changes in a_0, a_1, a_2 and b_1. So as *P* and *Q* vary over their prescribed intervals, Equation 1 describes a family of plants rather than a nominal plant. Since a_0, a_1, a_2 and b_1 depend simultaneously on ρ, this family of plants can approximated by a subset of the following interval plant:

$$G_p(s) = \frac{[\underline{b}_1 \quad \overline{b}_1]s}{[\underline{a}_4 \quad \overline{a}_4]s^4 + [\underline{a}_3 \quad \overline{a}_3]s^3 + [\underline{a}_2 \quad \overline{a}_2]s^2 + [\underline{a}_1 \quad \overline{a}_1]s + [\underline{a}_0 \quad \overline{a}_0]} \tag{3}$$

Robust stability of this interval plant implies that of the family of plants. However, instability of such interval plant does not imply instability of such family of plants. The bounds of each coefficient in this interval plant can easily be computed using Definition 3 in Sec. III. Stability of interval plants is often studied via Kharitonov theorem [12].

III. Interval Mathematics Perliminaries

Interval arithmetic was originally developed parallel to conventional arithmetic [13-14]. Interval analysis considers each number as an interval rather than a fixed point. An interval number is the set of real numbers x such that $\underline{x} \le x \le \overline{x}$. Let $\mathbf{R}$ denotes the set of all real numbers, then $x = [\underline{x} \quad \overline{x}] := \{\hat{x} \in \mathbf{R} \mid \underline{x} \le \hat{x} \le \overline{x}\}$, where $\underline{x}, \overline{x}$ are elements of $\mathbf{R}$ with $\underline{x} \le \overline{x}$ and $\hat{x}$ is the generic element $\hat{x} \in x$, further we can write $x \in \mathbf{IR}$ where the notation $\mathbf{IR}$ stands for an interval over real numbers. If upper and lower bounds are equal, interval arithmetic is reduced to conventional arithmetic.

Definition 1: Let $[\underline{a} \quad \overline{a}]$, $[\underline{b} \quad \overline{b}] \in \mathbf{IR}$, then

$$[\underline{a} \quad \overline{a}] + [\underline{b} \quad \overline{b}] = [\underline{a} + \underline{b} \quad \overline{a} + \overline{b}]$$

$$[\underline{a} \quad \overline{a}] - [\underline{b} \quad \overline{b}] = [\underline{a} - \overline{b} \quad \overline{a} - \underline{b}]$$

$$[\underline{a} \quad \overline{a}] \cdot [\underline{b} \quad \overline{b}] = [\min(\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}) \quad \max(\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b})]$$

$$[\underline{a} \quad \overline{a}] / [\underline{b} \quad \overline{b}] = [\underline{a} \quad \overline{a}] \cdot [1/\overline{b} \quad 1/\underline{b}],\ 0 \notin [1/\overline{b} \quad 1/\underline{b}].$$

Clearly, division is not defined if $0 \in [1/\overline{b} \quad 1/\underline{b}]$.

Definition 1 makes clear that subtraction and division in interval arithmetic are not the inverse operation of addition and multiplication. Some of the properties of interval operations are summarized and presented in Definition 2.

Definition 2: Let $[\underline{a} \quad \overline{a}]$, $[\underline{b} \quad \overline{b}], [\underline{c} \quad \overline{c}] \in \mathbf{IR}$, then

$$[\underline{a} \quad \overline{a}] + [\underline{b} \quad \overline{b}] = [\underline{b} \quad \overline{b}] + [\underline{a} \quad \overline{a}]$$

$$[\underline{a} \quad \overline{a}] + ([\underline{b} \quad \overline{b}] + [\underline{c} \quad \overline{c}]) = ([\underline{a} \quad \overline{a}] + [\underline{b} \quad \overline{b}]) + [\underline{c} \quad \overline{c}]$$

$$[\underline{a} \quad \overline{a}] \cdot [\underline{b} \quad \overline{b}] = [\underline{b} \quad \overline{b}] \cdot [\underline{a} \quad \overline{a}]$$

$$[\underline{a} \quad \overline{a}] \cdot ([\underline{b} \quad \overline{b}][\underline{c} \quad \overline{c}]) = ([\underline{a} \quad \overline{a}][\underline{b} \quad \overline{b}]) \cdot [\underline{c} \quad \overline{c}]$$

$$[\underline{a} \quad \overline{a}] \cdot ([\underline{b} \quad \overline{b}] + [\underline{c} \quad \overline{c}]) \subseteq [\underline{a} \quad \overline{a}] \cdot [\underline{b} \quad \overline{b}] + [\underline{a} \quad \overline{a}] \cdot [\underline{c} \quad \overline{c}]$$

$$\gamma \cdot ([\underline{a} \quad \overline{a}] + [\underline{b} \quad \overline{b}]) = \gamma \cdot [\underline{a} \quad \overline{a}] + \gamma \cdot [\underline{b} \quad \overline{b}], \quad \forall \gamma \in \mathbf{R}$$

Definition 2 describes the properties of commutativity, associativity and sub-distributivity of interval numbers.

Definition 3: If $f(x)$ is unary operation on $\mathbf{R}$, then $f([\underline{x} \quad \overline{x}]) = [\min_{x\in[\underline{x} \quad \overline{x}]} f(x) \quad \max_{x\in[\underline{x} \quad \overline{x}]} f(x)]$.

Now let us discus robust stability of a dynamical system using IA. Consider that the system Σ depends on an n_p-dimensional time invariant parameter vector ρ as follows:

$$\Sigma(\rho): \dot{x}(t) = A(\rho)x(t) + B(\rho)u(t), \ \ y(t) = C(\rho)x(t) \tag{4}$$

where ρ belongs to the box $[\rho]$. Let $\Sigma([\rho])$ be the set of all systems $\Sigma(\rho)$ such that ρ belongs to $[\rho]$. $\Sigma([\rho])$ is robustly stable if and only if $\Sigma(\rho)$ is stable for any ρ in $[\rho]$. Proving robust stability of $\Sigma([\rho])$ is one of the fundamental topics of robust control theory [12]. The set of all the characteristic polynomials associated with $\Sigma([\rho])$ is defined by

$$\Delta(s,[\rho]) := \{a_n(\rho)s^n + a_{n-1}(\rho)s^{n-1} + ... + a_0(\rho) \mid \rho \in [\rho]\} \tag{5}$$

Let $\mathbf{a}(\rho) := [a_n(\rho) \quad a_{n-1}(\rho) \quad \cdots \quad a_1(\rho) \quad a_0(\rho)]$ denotes the coefficient function and the coefficient set by $\mathbf{A} := \{\mathbf{a}(\rho) \mid \rho \in [\rho]\} = \mathbf{a}([\rho])$. A polynomial $\Delta(s,\rho)$ is completely specified by its coefficient function $\mathbf{a}(\rho)$. As result, $\mathbf{a}(\rho)$ is used to designate the polynomial $\Delta(s,\rho)$ and $\mathbf{A}$ is used to designate the corresponding set of polynomials. Consider a family $\mathbf{A}$ of polynomials, and assume the degree of each is equal to n. $\mathbf{A}$ is robustly-stable if and only if all polynomials in $\mathbf{A}$ are stable. The problem to be considered now is the test of robust stability for different types of $\mathbf{A}$.

Case 1: the coefficient set $\mathbf{A}$ is a *box*, i.e.,

$$\mathbf{A} = [\underline{a}_n \quad \overline{a}_n] \times [\underline{a}_{n-1} \quad \overline{a}_{n-1}] \times ... \times [\underline{a}_1 \quad \overline{a}_1] \times [\underline{a}_0 \quad \overline{a}_0] \tag{6}$$

Thus the corresponding family of polynomials is classically termed as an interval polynomial given by:

$$\mathbf{A} = [\underline{a}_n \quad \overline{a}_n]s^n + [\underline{a}_{n-1} \quad \overline{a}_{n-1}]s^{n-1} + ... + [\underline{a}_1 \quad \overline{a}_1]s + [\underline{a}_0 \quad \overline{a}_0] \tag{7}$$

Case 2: $\mathbf{A}$ is a polytope, or equivalently designates a polytope of polynomials that meet with convexity.

Case 3: $\mathbf{A}$ is the image of a box $[\rho]$ by a function $\mathbf{a}(\cdot)$ and thus $\mathbf{A}$ designates an image-set polynomial.

Cases 1 &3 are considered in Sec IV to carry out the design.

IV. PD-BASED PSS DESIGN VIA INTERVAL ANALYSIS

Since PD controller looks like phase lead compensator functionally, it is selected in this study to accomplish the design of a robust PSS. The proposed design algorithm aims to compute the set of all robust stabilizing PD controllers that guarantee system robust stability over the following operating ranges:

$$P \in [0.2 \quad 1.0], \ \ Q \in [-0.2 \quad 0.5] \tag{8}$$

According to [5], these ranges encompass all practically operating conditions. The characteristic polynomial of the closed loop system using such a PD controller is expressed as follows:

$$a_4 s^4 + a_3 s^3 + (a_2 + b_1 K_d)s^2 + (a_1 + b_1 K_p)s + a_0 = 0 \tag{9}$$

Controller's parameters are then computed using interval polynomial and image-set polynomial methods. The test power system is borrowed from [6].

A. Interval Polynomial Method

The bounds of all coefficients in the characteristic polynomial are computed over the specified range of P and Q using Definition 3 which results in the following positive intervals:

$a_4 = [1 \quad 1], a_3 = [20.463 \quad 20.463], a_2 = [22.413 \quad 87.206], a_1 = [131.52 \quad 792.93], a_0 = [569.89 \quad 1763.7], b_1 = [2.439 \quad 11.574]$

$$\begin{aligned}&[\underline{a}_4 \quad \overline{a}_4]s^4 + [\underline{a}_3 \quad \overline{a}_3]s^3 + [\underline{a}_2 + \underline{b}_1 K_d \quad \overline{a}_2 + \overline{b}_1 K_d]s^2 \\ &\qquad + [\underline{a}_1 + \underline{b}_1 K_p \quad \overline{a}_1 + \overline{b}_1 K_p]s + [\underline{a}_0 \quad \overline{a}_0] = 0\end{aligned} \tag{10}$$

Interval Routh-Hurwitz array is created for interval polynomial (10) as follows:

$$\begin{array}{c|lll} s^4 & [\underline{a}_4 \quad \overline{a}_4] & [\underline{a}_2 + \underline{b}_1 K_d \quad \overline{a}_2 + \overline{b}_1 K_d] & [\underline{a}_0 \quad \overline{a}_0] \\ s^3 & [\underline{a}_3 \quad \overline{a}_3] & [\underline{a}_1 + \underline{b}_1 K_p \quad \overline{a}_1 + \overline{b}_1 K_p] & \\ s^2 & r_{21} & [\underline{a}_0 \underline{a}_3 / \overline{a}_3 \quad \overline{a}_0 \overline{a}_3 / \underline{a}_3] & \\ s^1 & r_{11} & & \\ s^0 & r_{01} & & \end{array}$$

where:

$$\begin{aligned} r_{21} = [&(\underline{a}_3 \underline{a}_2 + \underline{a}_3 \underline{b}_1 K_d - \overline{a}_4 \overline{a}_1 - \overline{a}_4 \overline{b}_1 K_p) / \overline{a}_3 \\ &(\overline{a}_3 \overline{a}_2 + \overline{a}_3 \overline{b}_1 K_d - \underline{a}_4 \underline{a}_1 - \underline{a}_4 \underline{b}_1 K_p) / \underline{a}_3 \ ] \end{aligned}$$

$$\begin{aligned} r_{11} = [&(\underline{a}_3 \underline{a}_2 + \underline{a}_3 \underline{b}_1 K_d - \overline{a}_4 \overline{a}_1 - \overline{a}_4 \overline{b}_1 K_p)(\underline{a}_1 + \underline{b}_1 K_p) / \overline{a}_3 - \overline{a}_0 \overline{a}_3^{\,2} / \underline{a}_3 \\ &(\overline{a}_3 \overline{a}_2 + \overline{a}_3 \overline{b}_1 K_d - \underline{a}_4 \underline{a}_1 - \underline{a}_4 \underline{b}_1 K_p)(\overline{a}_1 + \overline{b}_1 K_p) / \underline{a}_3 - \underline{a}_0 \underline{a}_3^{\,2} / \overline{a}_3] \end{aligned}$$

$$r_{01} = [\underline{a}_0 \underline{a}_3 / \overline{a}_3 \quad \overline{a}_0 \overline{a}_3 / \underline{a}_3]$$

If $\underline{r}_{21}, \underline{r}_{11}, \underline{r}_{01}$ denote lower bounds of the intervals r_{21}, r_{11}, r_{01} respectively, then *PD-controller* can robustly stabilize this interval polynomial *iff* $\underline{r}_{21}, \underline{r}_{11}, \underline{r}_{01} > 0$. Obviously, $\underline{r}_{11} > 0$ implies $\underline{r}_{21} > 0$ and hence it suffices to make $\underline{r}_{11} > 0$, i.e.,

$$(\underline{a}_3 \underline{a}_2 + \underline{a}_3 \underline{b}_1 K_d - \overline{a}_4 \overline{a}_1 - \overline{a}_4 \overline{b}_1 K_p)(\underline{a}_1 + \underline{b}_1 K_p) / \overline{a}_3 - \overline{a}_0 \overline{a}_3^{\,2} / \underline{a}_3 > 0$$

$$\begin{aligned} K_d > (\overline{a}_0 \overline{a}_3^{\,3} / \underline{a}_3^{\,2} \underline{b}_1) / (\underline{a}_1 + \underline{b}_1 K_p) + (\overline{a}_4 \overline{b}_1 / \underline{a}_3 \underline{b}_1) K_p \\ + (\overline{a}_4 \overline{a}_1 - \underline{a}_3 \underline{a}_2) / (\underline{a}_3 \underline{b}_1) \end{aligned}$$

Since $\underline{a}_3 = \overline{a}_3 = a_3$ and $\underline{a}_4 = \overline{a}_4 = a_4$, the boundary of the stability region in K_p-K_d parameter-plane is calculated as:

$$K_d^{cr} = \frac{\overline{a}_0 a_3 / \underline{b}_1}{\underline{a}_1 + \underline{b}_1 K_p} + \frac{\overline{a}_1 a_4 - \underline{a}_2 a_3}{a_3 \underline{b}_1} + \left(\frac{a_4 \overline{b}_1}{a_3 \underline{b}_1}\right) K_p \tag{11}$$

Assuming $k_p \in [0 \quad 500]$, the boundary of the stability region can be plotted as shown in Figure 2. Minimum value of the derivative gain that ensures robust stability is precisely computed as $K_{d\min}^{cr} = 69.22$ which is large. Interval RH array provides only a sufficient condition for stability. This is due to the fact that in performing the interval operations, the intervals are implicitly over-bounded. Accordingly, (11) presents a fast but conservative boundary.

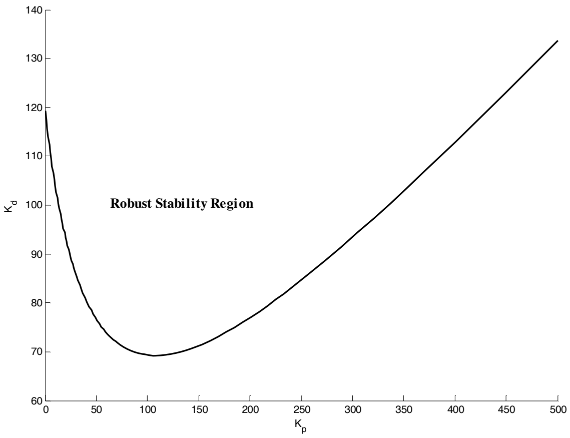

Figure 2 Region of robust stability in K_p-K_d parameter plane using interval polynomial method

B. Image-Set Polynomial Method

To relax the equality constraint (11), an image-set polynomial method is considered. The image-set case of the closed loop characteristic polynomial is given as follows:

$$\begin{aligned} a_4(\rho)s^4 + a_3(\rho)s^3 &+ [a_2(\rho) + b_1(\rho)K_d]s^2 \\ &+ [a_1(\rho) + b_1(\rho)K_p]s + a_0(\rho) = 0 \end{aligned} \quad (12)$$

Putting $\mathbf{a_i} = a_i(\rho),\ i = 0,1,...,4$ and $\mathbf{b_1} = b_1(\rho)$ then construct the classical RH array as follows:

$$\begin{array}{c|lll} s^4 & \mathbf{a_4} & \mathbf{a_2} + \mathbf{b_1}K_d & \mathbf{a_0} \\ s^3 & \mathbf{a_3} & \mathbf{a_1} + \mathbf{b_1}K_p & \\ s^2 & \mathbf{r_{21}} & \mathbf{a_o a_3} & \\ s^1 & \mathbf{r_{11}} & & \\ s^0 & \mathbf{a_o a_3} & & \end{array}$$

where: $\mathbf{r_{21}} = \mathbf{a_2 a_3} + \mathbf{a_3 b_1}K_d - \mathbf{a_1 a_4} - \mathbf{a_4 b_1}K_p$,

$$\mathbf{r_{11}} = (\mathbf{a_2 a_3} + \mathbf{a_3 b_1}K_d - \mathbf{a_1 a_4} - \mathbf{a_4 b_1}K_p)(\mathbf{a_1} + \mathbf{b_1}K_p) - \mathbf{a_0 a_3^2}$$

Noticeably, $\mathbf{r_{11}} > 0$ implies that $\mathbf{r_{21}} > 0$ and hence it is sufficient to make $\mathbf{r_{11}} > 0$, i.e.,

$$K_d > (\mathbf{a_4/a_3})K_p + (\mathbf{a_1 a_4} - \mathbf{a_2 a_3})/(\mathbf{a_3 b_1}) + (\mathbf{a_0 a_3/b_1})/(\mathbf{a_1} + \mathbf{b_1}K_p)$$

Robust stability of the family of polynomials is guaranteed *iff* $K_d > \max\limits_{P \in [\underline{P}\ \ \overline{P}], Q \in [\underline{Q}\ \ \overline{Q}], K_p = K_p^*} R^*$, where R^* is given as follows:

$$R^* = (\mathbf{a_4/a_3})K_p^* + (\mathbf{a_1 a_4} - \mathbf{a_2 a_3})/(\mathbf{a_3 b_1}) + (\mathbf{a_0 a_3/b_1})/(\mathbf{a_1} + \mathbf{b_1}K_p^i)$$

The exact boundary of robust stability region can be computed using the following "max" equality constraint:

$$K_d^{cr} = \max_{P \in [\underline{P}\ \ \overline{P}], Q \in [\underline{Q}\ \ \overline{Q}], K_p = K_p^*} R^* \quad (13)$$

Such a boundary is depicted in Figure 3 for $k_p \in [0 \quad 500]$. It is noticed that $K_{d\min}^{cr} = 4.52$ which is much smaller than that obtained by interval-polynomial method.

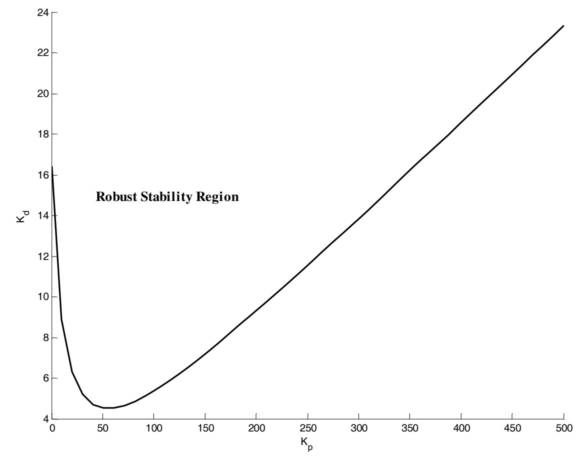

Figure 3 Region of robust stability in K_p-K_d parameter-plane using image-set polynomial method

V. Simulation Results

Figures 2 and 3 present the main result of this study. The equality constraint (11) gives fast but a conservative result, while (13) gives the exact result. Conservative results of the first method can be mirrored by observing the minimum allowable value of K_d that ensures robust stability in both methods.

A. Applicant controller validation based on linear model

An applicant controller is selected as K_p=41.6 and K_d=9.5. Figure 4 shows the effectiveness of such controller, to guarantee robust stability over (8). Damping factors of the dominant roots for a fine grid of 1024 plants are illustrated. Remarkably, the minimum damping factor greater than 0.6 sec^{-1} is achieved for the entire family of plants.

B. Validation based on nonlinear model

The system nonlinear model is simulated using the proposed controller. The response of the closed loop system due a 10% step change in mechanical torque is shown in Figure 5. The proposed design is compared to the conventional design presented in [15]. Remarkably, the proposed design ensures system stability at this test point while the conventional PSS fails. Poor performance of CPSS is accounted for by the fact that deviation from nominal point becomes significant.

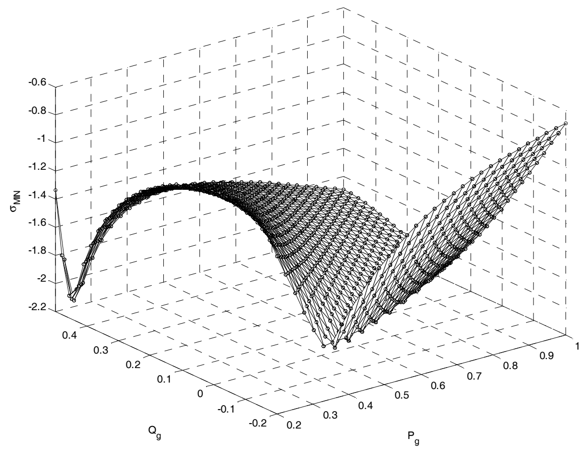

Figure 4 Minimum damping factors for 1024 plants within (8)

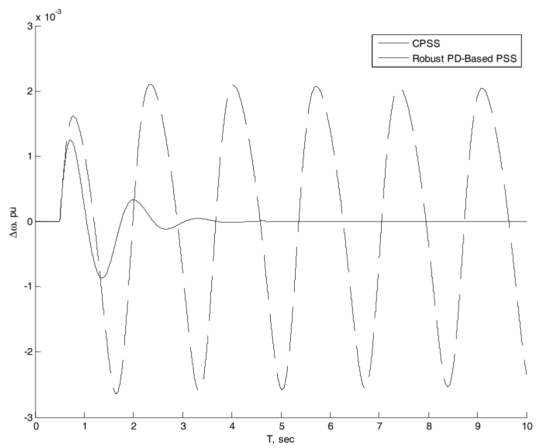

Figure 5 Rotor speed deviation due to 10% step change in mechanical torque: P_g=1.0, Q_g=-0.18pu.

C. Discussion and Comments

The proposed technique has been applied to a single-machine infinite-bus system equipped with a PD-based PSS. These simple models are initially used to demonstrate the principles of the proposed technique. However, further research is currently under way to extend the application to multimachine system where each machine is equipped with a three term controller. Such design can be applied to multimachine system by design a PSS for each machine at a time and considering the rest of the system as infinite bus. The resulting control is decentralized and local as it utilizes the speed deviation of the machine on which it is installed.

VI. Conclusion

This paper addressed the design of robust PSSs via interval arithmetic. Characterization of all robust stabilizing PD-based PSSs is presented analytically and numerically using two different methods namely interval polynomial and image-set polynomial. Interval Routh-Hurwitz array is considered to study the stability of an interval polynomial that capture all uncertainties due to loading conditions in the considered system. This procedure leads a fast and easily-computed robust stability constraint, but it results in a conservative stability boundary. The second method is basically a numerical one and results in an accurate boundary of robust stability region. The effectiveness of a sample controller is tested by plotting the dominant closed loop poles of 1024 plants that cover all the operating range. Moreover, simulation of the closed loop nonlinear model confirms the validity of the proposed controller. Using interval analysis over a set of compact real intervals can be a promising tool for characterizing all controllers for a certain plant via a set of nested loops assuming any controller structure.

VII. Refernces

[1] F. P. DeMello and C. Concordia, “Concepts of synchronous machine stability as affected by excitation control,” IEEE Trans. Power Apparatus and Systems, vol. 88, no. 4, pp. 316-329, April 1969.

[2] P. W. Sauer and M. A. Pai, Power System Dynamics and Stability, Prentice-Hall, Inc., 1998.

[3] E. V. Larsen and D. A. Swann, “Applying power system stabilizers: Parts I-III,” IEEE Trans. Power Apparatus and Systems, vol. 100, no. 6, pp. 3017-3046, June 1981.

[4] S. Chen and O. P. Malik, “H_∞ optimization-based power system stabilizer design,” IEE Proc. Generation, Transmission and Distribution, vol. 142, no. 2, pp. 179-184, 1995.

[5] P. S. Rao and I. Sen, “Robust tuning of power system Stabilizers using QFT,” IEEE Trans. on Control Sys. Technology, Vol. 7, No. 4, pp. 478-486, 1999.

[6] H. M. Soliman, A.L. Elshafei, A. Shaltout, and M. F. Morsi, “Robust Power System Stabilizer,” IEE Proc. Generation, Transmission and Distribution, vol. 147, no. 5, pp.285–291, Sept. 2000.

[7] H. Werner, P. Korba, and T. Chen Yang, “Robust tuning of power system stabilizers using LMI techniques,” IEEE Trans. Control Systems Technology, vol. 11, no.1, pp. 147-2003, Jan 2003.

[8] Y.L. Abdel-Magid, and M.A. Abido, “Optimal multiobjective design of robust power system stabilizers using genetic algorithms,” IEEE Trans. Power Systems, vol. 18, no. 3, pp. 1125-1132, Aug. 2003.

[9] Z. Wang and F. L. Alvarado,” interval arithmetic in power flow analysis,” IEEE Trans. On Power Sys., Vol. 7, No. 3, pp. 1341-1344, Aug. 1992.

[10] A. Vaccaro, C. A. Canizaros and K. Bhattacharyya, “A range arithmetic based optimization model for power flow analysis under interval uncertainty,” IEEE Trans. On Power Sys., Vol. pp, No. 99, pp. 1- , 2012.

[11] C. Rakpenthai, S. Uatrongjit and S. Premrudeeprechacharn, “State estimation of power system considering network parameters uncertainty based on parametric interval linear systems,” IEEE Trans. on Power Sys., Vol. 27, No. 1, pp. 305-313, 2012.

[12] S. P. Bhattacharyya, H. Chapellat, and L. H. Keel, Robust Control: the parametric approach, Prentice-Hall, 1995.

[13] R. E. Moore, Interval Analysis, Prentice-Hall, Cliffs, NJ., 1966

[14] L. Jaulin, M. Kieffer, O. Didrit and E. Walter, Applied Interval Analysis, Springer-Verlag, London, 2001.

[15] T. C. Yang, “Applying H_∞ optimization method to power system stabilizer design Part I: single machine infinite bus systems,” Electrical Power &Energy Systems, vol. 19, no. 1, pp. 29-35, Jan. 1997.